Survey Team 3

Teachers' Collective Work as Regular School Practice for Teacher Professional Development and Learning

Birgit Pepin[1], Zeger-jan Kock[1,2], Hiro Ninomiya[3], Yudong Yang[4],
Bill Atweh[5], Gerard Sensevy[6], and Jehad Alshwaikh[7],

ABSTRACT   In this report we present the results from an extensive search of the literature regarding mathematics teachers' collective work in schools, in the Eastern and Western literature. In particular, we try to answer the research questions related to the following themes: (1) the nature of mathematics teachers' collective work as regular school practice; (2) the participants of such school-based collective work and their roles; and (3) the professional development and learning that can be observed in school-based teacher collective work. In terms of theoretical frames, results show that different variations of Lesson Study have been the main frame for teachers' collective work at school level, in particular of course in Japan and China, but also increasingly in Western countries such as the UK, the Netherlands, Portugal and Spain. The choice of this frame also impacted on the nature of the collective work: working in cycles of lesson (and learning progression) planning, enactment, and evaluation, leading to the re-design of lessons. Whilst in Western countries participants comprised a mix of teachers and researchers, in most Eastern countries, teachers would also work on their own (as a group of teachers) or with so-called expert teachers in their collective groups. Teacher learning resulting from collective work was reported in terms of: (a) lesson planning and preparation; (b) pedagogical content knowledge(c) classroom practices; (d) general pedagogy; (e) social and personal issues in the mathematics classroom. The findings have implications for the conceptualization of school-based teacher collective work, for the support and facilitation of such work, and for research, in particular in terms of teacher agency.

Keywords: Teacher collaborative work; Regular school practice; Teacher professional development; Teacher learning; Lesson Study.

[1] Eindhoven University of Technology, Eindhoven, the Netherlands. E-mail: b.e.u.pepin@tue.nl
[2] Fontys University of Applied Science, Tilburg, the Netherlands. E-mail: z.kock@fontys.nl
[3] Saitama University, Japan. E-mail: hiro2001@mail.saitama-u.ac.jp
[4] Shanghai Academy of Educational Sciences, Shanghai, China. E-mail: yydong@cnsaes.org.cn
[5] Consultant, Australia. E-mail: b.atweh@oneworldripples.com
[6] University of Brest, Rennes, France. E-mail: gerard.sensevy@espe-bretagne.fr
[7] Beirut University, Lebanon. E-mail: jalshwaikh@gmail.com

1. Introduction

During the past decades, teacher collaboration has received increasing attention from both the research and the practice fields. It has been claimed that teacher collaboration can positively influences the whole school community. DuFour et al. (2005) contend that collaborative learning communities "hold out immense, unprecedented hope for schools and the improvement of teaching" (p. 128). Amongst others, teacher self-efficacy has been found to have improved (e.g., Puchner & Taylor, 2006), increased teaching effectiveness (e.g., Graham, 2007), and improvement of instructional quality (e.g., Jackson & Bruegmann, 2009; Hochweber et al., 2012). These positive effects will improve their quality as professionals and as Hattie (2003) suggests, teacher quality alone accounts for 30% of the variance in student performance. Hattie (2015) also claims that teacher collaborative working communities will enhance teacher effectiveness and expertise. Moreover, selected research has shown that the positive influence of teacher collaboration transcends the teacher community, and it has been suggested that professional collaborative activities might have a positive effect on student achievement (e.g., Dumay et al 2013; Goddard et al. 2010).

Whilst in many (Western) countries previously teacher professional development activities were mainly conducted at and by universities and teacher education institutions, nowadays they are often run by local or regional school boards and agencies at school level. This trend goes hand-in-hand with proposals that teachers become partners in the design of their curriculum, rather than 'simply' implementing the curriculum, supported by (government) approved textbooks. Moreover, due to the availability of an enormous amount of free educational resources on the web, teachers ask for guidance and support for choosing and appropriating those resources for their classroom, and the closest support lies at school level, with their colleagues (in their or neighbouring schools). However, this trend also asks for teacher agency, their professional agency: e.g., their decisions to participate in or withdraw from the teacher collective; which resources to ask for and use; which foci to choose in the collective; how to collaborate with colleagues/peers; which role to take in the collective.

At the same time teachers' collective work as regular school practice has a long history in many (mainly Eastern) countries: Lesson Study in Japan and Teaching Research Groups in China are well known examples. However, varying forms of such practice exist in many countries and in varying educational contexts. Over time, and particularly in more recent years, these practices have been shared and researched leading to the evolution of a wide, yet dispersed, knowledge base.

In this paper, we present the results from our international survey of the literature regarding mathematics teachers' collective work in schools. In particular, we try to answer the following research question:

> *What can be learnt from an examination of common features of mathematics teachers' collective work as regular school practice as well as from variations in practices and their rationales in different national contexts?*

We ask the following sub-questions:

1) *What is the nature of mathematics teachers' collective work as regular school practice, and how does this relate to situation, culture and context?*
2) *Who is engaged in such school-based collective work, what are the roles of those people involved, and how do they relate to each other in the different communities?*
3) *What kinds of learning can be observed in school-based teacher collective work? (How does teacher collective learning happen in teacher collectives at school, what is the evidence for their learning?)*

In the following (second) section, we present the background to the present study. This will be followed, in the third section, by the methods we used to conduct the literature review. In the fourth section, we explain the theoretical frames. In the fifth section we present the results relating to the research sub-questions, and in last section (section six) we refer to the main research question with our conclusions.

2. Background

In the previous section we have provided a rationale for conducting the survey we did, as we contend that teacher collaborative work is an important area for study. In earlier and recent ICME- related studies this has been acknowledged by: e.g. Borko and Potari 2020 (ICMI-25 Study); Robutti et al. 2016 (ICME Survey 13); Jaworski et al. 2016; Adler et al. 2005 (ICME 2004 survey). We build on this body of work, and attempt to establish in which ways teachers' collaborative work at school level has developed and how it varies across contexts.

3. Methods

To identify the relevant literature to answer our research questions, we conducted a systematic literature review. This review mainly relied on the procedures of a thematic synthesis (Xiao & Watson, 2019) with the overarching aim to build on the current body of literature, to summarize what is known about teachers' collective work at school level. Since research on teachers' collective work has enormously increased over recent years, we reduced our literature review to publications that were published since 2015. We went through all titles and abstracts from the following list of journals, conference proceedings and books in order to identify the papers comprising recent research on mathematics teachers' collaborative work at school level:

- Journals:
    - ✓ Educational Studies in Mathematics
    - ✓ International Journal of Science and Mathematics Education
    - ✓ Journal of Mathematics Teacher Education, Research in Mathematics Education
    - ✓ ZDM — Mathematics Education; Mathematics Teacher Education and Development;
    - ✓ Professional Development in Education

4

- Mathematics Education Research Journal
- Journal of Science & Mathematics Education in South-East Asia
- Mathematics Teacher Education & Development; Mathematics Teacher
- The Montana Mathematics Enthusiast
- The Mathematics Educator
- International Journal of Lesson and Learning Study
- Teaching and Teacher Education
- South African Journal of Education
- Journal of Research in Mathematics Education
- Teacher Education Quarterly
- Professional Development in Education

- Conference Proceedings:
  - Congress of the European Society for Research in Mathematics Education (CERME),
  - Conference of the International Group for the Psychology of Mathematics Education (PME)
  - Mathematics Education Research Group of Australasia (MERGA)

- Books:
  - The International Handbook of Mathematics Teacher Education (Vol. 3, Participants in mathematics teacher education- individuals, teams, communities and networks; Krainer & Wood (Eds.) 2008)
  - The Resource Approach to Mathematics Education (Trouche et al., 2019)
  - Mathematics lesson study around the world (Quaresma et al. 2018)
  - Lesson study research and practice in mathematics education (Hart et al. 2011)

- National/regional publications (in the respective languages) in the following countries: Japan, China, Netherlands, Lebanon, Australia

- Reports by researchers from particular countries (based on selected questions): Italy, Denmark, Singapore, Israel.

Altogether we searched through 21 journals and 4 different conference proceedings between 2015 and 2021, to collect about 200 articles (including national publications in the native languages), in addition to reports from four countries. After identifying a corpus of the relevant literature, we went through the literature identifying results on mathematics teacher collective work at school level that related to the three research questions.

4. Theoretical Frames

The following theoretical frames are explained below:
- (1) teacher collective work (at school level);
- (2) Lesson Study;
- (3) teacher agency.

4.1. Teacher collective work at school level

Within mathematics teachers' collective work, we can basically distinguish between two types: (1) Lesson study (please see section 4.2) with its three distinct features: planning a research lesson collaboratively; conducting and observing the planned lesson; jointly reflecting on the lesson based on observations of student activity (Murata, 2011); and (2) teachers' collective work on a proposed (or agreed, e.g., by a project) theme: e.g., reasoning and proof in commonly used textbooks. This would not necessarily include lesson preparation or indeed the enactment of a planned lesson.

Another distinction of teachers' collective work relates to **the place of the collective work**: e.g., in school, at a distance (online), or at university. Clearly, we refer here only to work that is school-based. However, school-based teacher collective work can also be online, that is at a distance. The latter have enormously increased, possibly due to COVID related measures.

It can be said that most of the collective work at school level includes planning and enacting lessons, as this is the main part of teachers' daily work. Hence, we can say that most of mathematics teachers' collective work at school level relates to Lesson Study, one way or another, and this is the reason why we have emphasized this way of working in our theoretical frames.

In order to conceptualize lesson study adaptations (also in their home Danish context), Skott and Moeller (2020) have used the notion of *figured worlds* (Holland et al., 1998), asking 'What characterizes the dominant figured worlds when groups of teachers engage in lesson study in a Danish context?'. Their results and insights stress

> "the importance of working on adaptations of approaches such as lesson study in order to transform issues of culture and power in the teachers' local setting. This applies in particular to those related to the three characteristics of a Danish teaching culture identified earlier …: teacher methodological autonomy (as interpreted from a teaming perspective), teacher collaboration characterized by functionality of teaching and a family culture, and the tendency to shake off macro-level demands." (p. 8/9)

This importance was supported by parts of their data showing that some teachers occupying senior-teacher positions would alternate between "old-hand" and "development-oriented" positions. Hence, they conclude that in their (Danish context) "it is necessary to address these broader issues of culture and power in order to adapt lesson study in a Danish context" (p.9). From this study (and others), we conclude that it is indeed necessary to distinguish between (research on) **lesson study adaptations within and outside the East Asian region.**

In terms of Lesson Study adaptations, Ding and Jones (2020) compared three such adaptations/models, each designed for supporting (and studying) in-service teacher collaboration and learning: (1) The Action-Education Model (AE) (Gu & Gu, 2016), a combination of Keli study (exemplary lesson development) practiced by researchers and teachers in schools in China and action research; (2) Learning Study (LS) (Lo & Marton, 2012), a combination of Lesson Study and design study originally conducted

in Hong Kong; (3) The Community-Centered (CC) model for teacher learning (Borko et al., 2005), a university-based summer institute program for supporting mathematics teacher collaboration (and learning) in the United States. The authors note that both Lesson Study and Learning Study (LS) address simultaneously lesson plan design and implementation as a whole teacher learning process, and (referring to Huang and Shimizu, 2016) how theory can be used to guide teaching and how teaching experiments can further refine theory (p.115). Interestingly, the western design studies (e.g., Cobb et al. 2017) share this view: whilst practically supporting teachers in improving specific aspects of their instructional practice, theoretically, they aim at designing and evaluating (and possibly re-designing) learning progressions (in association with instructional practices) and the teacher learning that goes with it.

4.2. Lesson study

Lesson Study is a complex professional learning approach. Several researchers have used the metaphor of an iceberg to capture the unseen features of lesson study with respect to the task for exposing student thinking and impacting student learning. Their metaphor is useful, in as much as the iceberg has much beneath the surface, many of the features (or essentials) of lesson study are not immediately obvious, and exposing them is said to assure fidelity of implementation of those essential features (Hart et al. 2011).

Historically, Lesson study is a collaboration-based teacher professional development approach that originated in Japan (e.g., Fernandez & Yoshida 2004) and also in China. Over the past decade it has attracted the attention of an international audience: e.g., in 2002 it was one of the foci for the Ninth Conference of the International Congress on Mathematics Education (ICME).

Lesson study incorporates many characteristics of effective professional development programs identified in prior research: e.g., it is site-based, practice-oriented, focused on student learning, collaboration-based, and research-oriented (e.g., Borko 2004; Cochran-Smith & Lytle 2001; Darling-Hammond 1994). Lesson study places teachers at the center of the professional activity with their interests and a desire to better understand student learning based on their own teaching experiences. The idea is straight forward and authentic: teachers share a question/goal regarding their students' learning and they come together based on that question; they plan a lesson to make student learning visible, and examine and discuss what they observe. Through multiple iterations of the lesson design, refinement, enactment and collection of data on student learning, reflection on lesson, and re-design process, teachers have many opportunities to discuss student learning and how their teaching affects it. Lesson study typically has a research lesson (live lesson observation) as the centerpiece of the study process (e.g., Fernandez & Yoshida 2004; Wang-Iverson & Yoshida 2005). The main purpose of this step is not to plan a perfect lesson but to test a teaching approach (or investigate a question about teaching) in a live context to study how students learn. During lesson planning, teachers also have an opportunity to study curricular materials,

which may help teachers' content knowledge development. During the lesson, teachers attend to student thinking and take notes on different student approaches. In the discussion after the lesson, teachers discuss student learning based on the data they have collected during the observation (Murata 2011).

There are other professional development programs that incorporate many of the characteristics of lesson study (e.g., action research). For example, in China, the concept of Lesson Design Study has been known to work well in the Teaching Research Groups in China (Ding et al. 2019). And there are also many adaptations to Lesson Study, in particular in the United States (e.g., Amador & Carter, 2018) and in Europe (e.g. Manolino, 2020) However, what is typically different in Lesson Study is the *live* research lesson. This is said to create a unique learning opportunity for teachers. Shared classroom experiences, such as teacher noticing of certain aspects of teaching and learning, might not otherwise be shared.

4.3. Teacher agency

From the work on agency, agency is known to be related to social systems or individual characteristics: e.g., making choices among alternatives, taking initiative or being able to influence oneself and others; and is both constrained and afforded by social relations and structures, particularly power relations (e.g., Mercer 2011). Mercer (2011, p. 428) argues:

> humans as agents [are] able to influence their contexts, rather than just react to them, in a relationship of ongoing reciprocal causality in which the emphasis is on the complex, dynamic interaction between the two elements

At the same time, Etelapelto and her colleagues (2013) argue for a subject-centered, sociocultural view of professional agency, which takes the individual and social contexts of agency to be analytically separate but mutually constitutive (2013, p. 45). In understanding agency from this perspective, they say, we need to investigate:

> how agency is practiced and how it is resourced, constrained and bounded by contextual factors, including power relations and discourses, and further by the material conditions and cultures of social interaction (2013, p. 61).

The same group of researchers also argue that agency has a temporal aspect, in that people's life histories and prior experiences influence their agency in relation to their contexts (Etelapelto et al. 2013). Biesta et al. (2015) argue that agency is an emergent phenomenon of actor–situation relations and is something that people do, rather than have, i.e. agency is enacted in context and denotes the 'quality of engagement of actors with temporal-relational contexts-for-action' rather than a property, capacity or competence of the person (2015, p. 626). This means that agents act 'by means of their environment rather than simply in their environment' (2015, p. 626).

5. Results

In this section we answer the research sub-questions with data and examples from the literature review.

5.1. *What is the nature of mathematics teachers' collective work as regular school practice, and how does this relate to situation, culture and context?*

We now summarize insights from studies and reports of mathematics teacher collective work at school-level (1) within and (2) outside the Austral-Asian region, and (3) at school and (4) school-based but at a distance (online).

5.1.1 Austral-Asia

Starting with **Japan**, one of the major characteristics, or the nature, of Japanese mathematics teachers is their **voluntary in-service training**. Baba et. al. (2018) discusses the background of "mathematics education Lesson Study in Japan" from four perspectives; Historical, Community, Institutional, and Development Assistance. In the Community perspective, the following facts are highlighted. Some schoolteachers, who usually are excellent teachers, have had a chance to get long-term in-service training under the supervision of university researcher. After their training, these teachers returned to their school and became "Leader Teachers" in the school. They also play an important role in their district teachers' communities. School teachers often have voluntary workshop in their district communities, in which teachers discuss about mathematics materials (Kyozaikenkyu, or material research), preparations for their Lesson Study in their school, writing papers about the results of their in-service trainings, and so on. Such community or workshop is called **"Kenkyukai", or math teacher circles**.

At the same time, schools sometimes have opportunities to get funding for their in-school teacher trainings. Such projects are assigned by district education office, by prefectural education office, or sometimes by ministry of education, and usually done by the strong leadership of "Leader Teachers", who are not only the teachers who have had long-term in-service training, but also the teachers who actively attend to "Kenkyukai", or math teacher circle activities. In this sense, Japanese Teachers' Collective Work as a Regular School Practice is done with strong implicit support of "Kenkyukai", or math teacher circles.

Another major characteristic in Japanese education is the existence of "Fuzoku schools", which are attached schools to university. Especially, Fuzoku schools attached to faculty of education (or university of education) have had special role in Japanese education. It is said that the major role of Fuzoku school are: 1) education to students just as regular school, 2) preservice teacher training, 3) practical study. Regarding (1), Fuzoku teachers are also schoolteachers who do the same work as other schoolteachers. Concerning (2), Fuzoku schools are the place for prospective teachers

to do their student teaching. Regarding (3), each Fuzoku school usually has its own "research or study topics", and play an important role to serve practical information about education. Mathematics teachers who are working for Fuzoku schools are usually the "Leader Teachers" in their math education communities. Obviously, they have more opportunities to write reports about their practical work in mathematics education. The number of Fuzoku schools attached to faculty of education is very small. There are about 70 Fuzoku within 20'300 elementary schools, 71 Fuzoku within 11'000 Jr. high schools, and 15 Fuzoku within 5000 senior high schools.

In a paper by Isoda (2020), the author reports on the historical development of Japanese Lesson Study. The author briefly sketches the Japanese theories for designing and reproducing better lessons to share and transfer the challenges and experiments of lesson study. Whilst Lesson study was initiated in 1873, it developed over more than a century, whilst "the major theories of mathematics education for designing and reproducing sciences were developed on the elaboration of theories proposed by various lesson study groups." (p.15) At the time of the author's writing, these can be summarized as the theories for: "clarify the objectives; distinguish teaching concept; establish the task sequence; and teaching approaches which includes assessments" (p. 15).

One of the differences of Lesson Study in Japan as compared to Western practices is that the importance of lesson preparation is largely underestimated in the West, and the collaborative work among teachers that goes into creating that lesson plan is largely under-appreciated by non-Japanese adopters of Lesson Study. This might be due to the effort involved being largely invisible to outsiders, with attention going to its most visible part, the live research lesson. The paper by Fujii (2016) makes visible "the process of lesson planning and the role and function of the lesson plan in Lesson Study" (p. 411). The paper identifies key features of the planning process in Lesson Study, including its focus on task design and the flow of the research lesson, and offers suggestions for educators seeking to improve Lesson Study outside Japan.

In **China**, mathematics teachers' collective work as regular school practice has been guaranteed by the teaching research system, because each mathematics teacher is 'naturally' (by default, as part of the job as a teacher)) a member of the mathematics TRG (Teaching Research Group) and LPGs (Lesson Preparation Group) in each school in mainland China. In *Secondary School Teaching Research Group Rule-book* issued by MOE in 1957, the duty of TRG was emphasized:

A Teaching Research Group is an organization to study teaching. It is not an administrative department. Its task is to organize teachers to do teaching research in order to improve the quality of education, but not to deal with administrative affairs (MOE, 1957).

Chinese Lesson Study (CLS) is just one of the forms of collective learning based on school-level TRG activities.

Not only mathematics teachers, every subject teacher belongs to a specific subject TRG for the reason of the teaching research system as the fundamental context in mainland China. Because most of the Chinese teachers who teach just one subject two

or three times a day, the same subject teachers are easily organized into subject-specific TRGs. This multi-tiered teaching research system is a network where province-level TRO oversee city-level TRO (see figure below), and city-level TRO oversee county-level TRO which oversee school-level TRGs (Yang, 2009; Yang & Ricks, 2013). The TRG is the basic unit in this network; its main responsibility is conducting research on teaching to solve the practical problems from teachers. So, mathematics teachers' collective work rooted deeply in the school-level TRG activities, which linked the lessons and the studies in their daily work (Fig. 1).
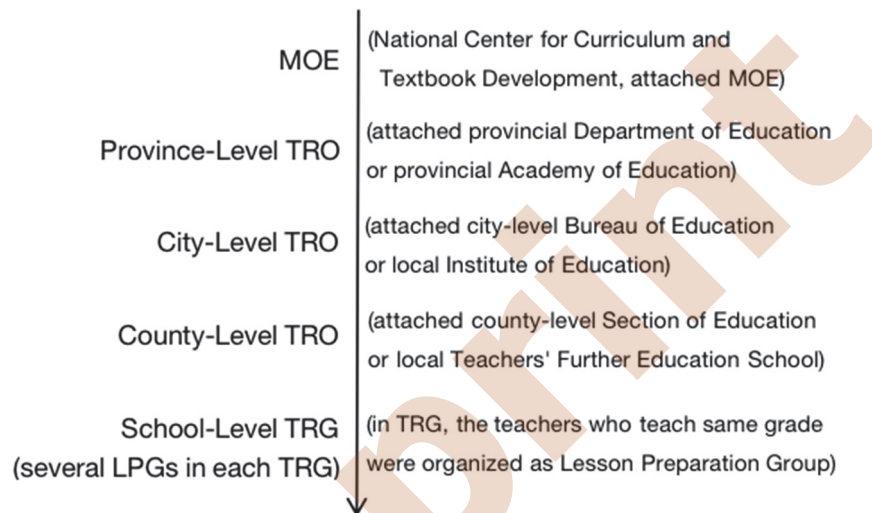

Fig. 1. The top to down guiding structure in Teaching Research System

In terms of content in such CLSs, a study by Huang et al. (2016) reports on student learning being studied by teachers, to improve teaching that promotes students' understanding. Interestingly, this CLS included didacticians (practice-based teaching research specialist and University-based mathematics educators) and mathematics teachers in China, who explored and documented how teacher participants "shifted their attention to students' learning by incorporating two notions of teaching: learning trajectory (LT) and variation pedagogy (VP)" (p.425). The former describes conjectured routes of children's thinking and learning with pertinent tasks to move towards the learning goals along the route, while the latter suggests strategies for using systematic tasks progressively. The concepts of LT and VP were used to guide planning, teaching, and debriefing throughout the LS process. Results revealed that "by building on the learning trajectory and by strategically using variation tasks, the lesson has been improved in terms of students' understanding, proficiency, and mathematical reasoning" (p.425). It was claimed that (and how) "theory-driven Lesson Study could help teachers improve their teaching and develop the linkage between theory and practice." (p.425)

In **Australia**, there has also been a particular interest in Japanese Lesson Study, as a vehicle to improve mathematics teaching practice. In their paper Groves et al. (2016) report on a small-scale research project, implementing structured problem-solving mathematics lessons through lesson study. The two major aims of the project were to investigate critical factors in the adaptation and effective implementation of (1) structured problem-solving mathematics lessons, and (2) Japanese Lesson Study as a model for teacher professional learning in the Australian context. Critical factors of Lesson Study were identified by the teachers as contributing to the success of the project. These included "the opportunities for in-depth lesson planning, the presence of large numbers of observers at the research lessons and the post-lesson discussions, and the insight provided by the knowledgeable other" (p. 501). Major constraints included the difficulty in finding suitable problem-solving tasks to match the Australian curriculum, and the teaching culture that emphasizes small-group rather than whole-class teaching.

Reporting on Lesson Study in **Korea**, Pang et al. (2016) describe how a lesson study using five practices for mathematics discussion was implemented in the Korean context. They contend that Lesson Study has had an effect on improving the quality of mathematics instruction and supporting teachers' professional development, in the sense that "the lessons were changed to specify learning goals for students, to devise mathematical tasks in a rigorous and meaningful way, and to design the lesson structure to maximize students' engagement" (p. 471).

5.1.2 *Europe and Middle-East*

During the past two decades, in Europe and Western countries (including North America) professional learning communities (PLCs) have been established, as they are seen as levers for teacher professional development. PLCs are generally defined as groups of teachers who come together to engage in regular, systematic and sustained cycles of inquiry-based learning, with the intention to develop their individual and collective capacity for teaching to improve student outcomes (Brodie 2021). PLCs are said to create spaces for ongoing, sustained professional development, in particular at school level, different from the often-fragmented professional development programs that many teachers are exposed to (Borko 2004, Cobb et al. 2018). PLCs can be seen as a special case of communities of practice (Wenger 1998), where members engage in professional learning (see section 5.3). One of the main intentions for PLCs is to deliberately position teachers as professional agents in their own professional development, through their making professional decisions as to what they need to do to enhance their teaching, in particular based on their understandings of their learners' needs. While much of the work on PLCs argues for teacher agency as a key driver of PLCs, it is not yet known what it means to develop teachers as agents and what teacher agency actually entails (Brodie 2021; Horn et al. 2018). However, the literature on PLCs converges on five key characteristics of successful PLCs (e.g., Brodie 2021): focus, long-term inquiry, collaboration, leadership support and trust. How these

characteristics play out in PLCs is central to their sustainability as spaces for professional development.

In Europe, these PLCs meet, for example, at school or at university, or in other commonly agreed spaces. In many European countries (e.g., UK) it is common to meet in school, on a voluntary basis. However, there is typically no institutionalized system of PLCs, as we see in China and Japan. Often, the PLCs are initiated by (European) projects and conducted by university academics in regional schools. In selected countries (e.g., NL), the PLCs (sometimes called DOTs- design teaching teams; see articles by Verhoef 2013, 2015) are initiated by national institutions to implement curriculum changes at school level. In recent years, Lesson Study in various forms (see earlier discussion; Skott & Moeller 2020; Ding & Jones 2021) has been promulgated as a suitable vehicle for professional development.

In Israel, Karsenty et al.'s (2019) team explored how secondary mathematics teachers, participating in a school-based video club (Sherin, et al. 2009) communicated with each other and with the facilitator along the different sessions of the club. Whilst there are different forms of video clubs, in this context a group of teachers met on a regular basis, usually under the guidance of a facilitator, to watch and discuss classroom video selected according to a certain aim. Analyzing their evaluative comments (with respect to the non- judgmental norms that this club aimed to nurture), three types of evaluative comments were identified, "reflecting varying degrees of teachers' capability to interpret and discuss observed teaching moves while attributing possible rationalizations to the filmed teacher's decisions." (p.3400) They found that as the club proceeded the communication became more productive.

In the UK, we found an example of Lesson Study in the context of the introduction of a New National Curriculum for Mathematics in England; this was not supported by a mathematics teacher educator (Warwick et al. 2016). They claim that Lesson Study is "rapidly becoming one of the most adopted models of teacher professional development worldwide" (p.555). They examined the teachers' discussions that were an integral part of the Lesson Study research cycle. In particular, they investigated "the 'dialogic mechanisms' that enable teachers' pedagogical intentions to be developed within the context of discussions that stem from observations of students as they address mathematical problems" (p.555). Findings suggested that a focus on student outcomes enabled teachers to collaborate effectively on developing pedagogical intentions to directly address student need.

Leaning on teacher collaboration for lesson planning, the paper by Pepin et al. (2017) reports on mathematics teachers re-designing their lessons due to curriculum changes in selected countries in Europe. Whilst the goal of this paper was to develop enhanced understandings of mathematics teacher design and design capacity when interacting with digital curriculum resources, it also offered new understandings of teacher collaboration in different context: e.g., France and Norway; small group collaboration (France) vs large group of teachers (Norway). Drawing on two different collective environments and two individual teacher cases working within these environments, the authors investigated and illustrated teachers' design processes (and

design capacity building) across a range of contexts and curriculum formations, with the focus on how digital resources can help to develop teacher design capacity.

In terms of teacher collaboration at the distance, we found several papers, all using different ways of communicating at a distance. One of these ways were MOOCs (e.g. Taranto et al. 2020). In this project the authors used two theoretical lenses (Meta-Didactical Transposition, Connectivism) to investigate teachers' learning processes (see also section 5.3). Results showed two different teachers' learning processes: one that evolved dramatically because of the interventions — they called it an 'explosion'; the other less proactively — they called it 'linear'. In the Danish context, as another example, Tamborg (2021) investigated how a national platform brought teachers together for professional development and how it affected teachers' work. The platform was the main tool to implement at scale "an evidence-based, objective-oriented approach to teaching". He concluded that the design of platforms conflicted with the needs of mathematics teachers.

5.1.3 Americas

Over the past decade, Lesson Study has become very popular, in particular in North America. Stigler and Hiebert (2016) reported that lesson study is gradually spreading around the globe, and that the Western community has "much to learn from how it is implemented in a variety of cultural contexts". (p. 581) They reflect on the goals of lesson study, the organizational supports required to sustain the practice in various contexts, and "the benefits that may be derived from making more explicit the connections between lesson study and the wider field of improvement science" (p.581). They claim that both research and practice can benefit from learning about and from such different practices.

To provide an example of such 'borrowing', Lewis (2016) presents a theoretical model of lesson study's impact on instruction by impacting on teachers' beliefs and their learning community, amongst others. She also describes four different types of lesson study in Japan: (1) incorporation of high-quality tasks and materials; (2) attention to processes that illuminate student thinking; (3) attention to system features; and (4) models for scale-up. (p.581) She points out their "synergies in producing a system where local teachers "demand" knowledge for their lesson study work and lesson study provides a collaborative, practice-based venue to try out recent innovations in curriculum and instruction" (p. 581).

In several Western countries (e.g., USA, France) we found teacher collaboration being established around how to make sense of new academic standards and how teachers may shape the implementation of those standards. In the US context, Johnson et al. (2016) reported on a study where professional development was organized around the analysis of mathematical tasks, to support teachers to prepare for standards implementation by helping them develop common understandings of standards and how to help students meet ambitious new learning goals. However, in reality designers and teachers brought different goals to the professional development context, which

became evident when teachers engaged in task analysis. Using a particular 'design tensions framework', they analyzed tensions within a research–practice partnership comprised of university researchers, district curriculum leaders, mathematics teachers, and Web engineers. Results showed the need for designers of professional development focusing on standards implementation, to be 'adaptive and willing to evolve activities to satisfy multiple stakeholders' goals for participation'.

In terms of distance learning in collectives, we found many ways of collaborating 'at school level'. For example, Larsen and Liljedahl (2017) used Twitter posts to analyze stimulating sustainable mathematics teacher collaboration in a 'distant professional development context'. To their surprise, an unprompted, unfunded, unmandated, and largely unstudied mathematics teacher community emerged where the mathematics teachers use social media to communicate about the teaching and learning of mathematics. Results indicated that enough redundancy and diversity among members is necessary to make conversations productive.

In summary, it can be said that the contexts and cultural education traditions influence the professional learning communities: their set-up, their practices, the tools used, and the expected outcomes. In the Western countries, many communities are driven by a desire to innovate or renew the curriculum and the pedagogy or to come to a meaningful integration of technology. These communities tend to be part of projects with a limited time frame. In the Eastern countries professional learning activities are more connected to the everyday teaching activities and focus on values and perceptions of 'good mathematics teaching' and 'good lesson planning'. In particular, countries like Japan and China have established a 'tradition' of teacher professional learning communities at school level.

5.2. Who is engaged in such school-based collective work, what are the roles of those people involved, and how do they relate to each other in the different communities?

Depending on the context (e.g., research project in Europe, Lesson Study in Japan, Learning Study in China), there are often different people involved in the collective work of teachers. As explained earlier, in the European context, teachers often work with teacher educators on Lesson Study or similar project that is most of the time financed by outside (school) funding bodies (e.g., EU financing), whilst in the Japanese and Chinese lesson study collaborations, classroom teachers (e.g., of the same grade) work together, sometimes with the support of expert teachers (Pepin et al. 2017). Despite these differences, a common thread running throughout the surveyed articles is the need for learning to be situated in collaboration with others. However, the collaboration can take on very different structures in supporting teachers' professional learning due to the different purposes and roles of the teachers, expert teachers or teacher educators in the studies (see 5.3).

Professional learning communities with mathematics teachers and teacher educators and/or expert teachers working and learning in collaborative groups show a

huge diversity of roles, identities and interactions. This makes it difficult to get an insightful overview of this diversity, to compare initiatives or to grasp the specificity of individual initiatives. In their article Krainer and Spreitzer (2020) selected seven recent articles (covering all continents) and analyzed them along the following dimensions: relevant actors, relevant targets, and relevant environments of the collaboration (RATE). In terms of 'relevant actors', they claim that (using the RATE scheme) apart from mathematics teachers, the seven articles show "a variety of actors", including teacher educators (6 initiatives), mathematicians (4), and policy makers (2). As social entities they found (video) "clubs", different "communities", (lesson study) "groups", (project) "partners" and (design and project) "teams" (p. 34).

Regarding Lesson/Learning Study in China, Gu et al. (2016) reported on the roles of experts and other participants. The team investigated how "mathematics teaching research specialists" mentor practicing teachers during post-lesson debriefs of a lesson study in China. Results of fine-grained analysis of post-lesson study debriefing revealed that the "Chinese teaching research specialists (expert teachers, see Pepin et al. 2017) pay a great deal of attention to practical knowledge, which consists of setting students' learning goals, designing instructional tasks, formative assessment of students' learning and improving instructional behaviors" and that "less attention is paid to mathematics content knowledge and general pedagogical knowledge" (p. 441). The teaching research specialists apparently also pay less attention to address issues raised by the teachers or to engage in dynamic dialogue with them. Using a purposefully-developed framework for analyzing mentoring activities emerges, the strengths and weaknesses of the teaching research specialists' mentoring strategies were identified.

It has been noted that in the USA, it is rare that teachers work with university colleagues in their school settings even though this collaboration often improves classroom instruction (Herrenkohl et al. 2010). Overall, university partnerships with teachers for professional development has been considered beneficial because of the potential of collaborative work in the teacher's own classroom to be relevant to practice. From this perspective, both teachers and researchers can draw on their own expertise and work as authentic partners. In a study by Jung and Brady (2016) in the USA, they investigated how a teacher and a researcher performed their roles when collaboratively implementing mathematical modeling tasks within a context of in situ professional development. The researcher–teacher partnership shown in this study demonstrated how such collaboration can be supported by sharing knowledge and resources (Lau & Stille 2014). Through this in situ professional development focusing on mathematical modeling tasks, "several teacher and researcher roles were highlighted: (1) the researcher's ways of opening the discussions and addressing the teacher's concerns, (2) the researcher's approaches to acknowledging the teacher's expertise, (3) the teacher's strategies for overcoming difficulties, and (4) the teacher's process of reflecting on the factors that helped student development" (p.291). While the teacher learned about the new mathematical modeling tasks and related research, she helped the researcher recognize classroom realities and implement modeling tasks in these realistic settings.

They also shifted roles at different stages of instructional practice (e.g., the researcher led classroom instruction or the teacher analyzed student work), which ensured that both teacher and researcher took ''the role of expert'' depending on the classroom situation (Lau & Stille 2014). The study supports the value and viability of this form of in situ professional development, indicating that significant changes in teachers' thinking (in this case about their students' mathematical model development) can occur in relatively short periods of time.

To summarize this section, it can be said that the different forms of professional learning communities include different actors. From the Chinese and Japanese cases of Lesson/Learning study, we learnt that these either include teachers (e.g., teaching the same grade) working on their own or with an expert, in a collaborative community. In these set-ups, the expert teachers (who can also be university teacher educators) are greatly appreciated, due to their seniority, their experience and expertise (e.g., Pepin et al., 2017). Teachers are expected to learn from the expert, perhaps even by 'imitating' the expert. In the European settings, university teacher educators often work with classroom teachers, not because of their seniority or teaching experience, but due to their knowledge about mathematics didactical theory — this is expected to 're-source' the teachers. However, in these settings teachers are expected to become involved in curriculum design (to take agency), including planning lessons and learning progressions, often according to newly implemented curriculum guidelines.

5.3. *What kinds of learning can be observed in school-based teacher collective work?*

In the studies we reviewed, teacher professionalization often has taken place in several dimensions. For example, teachers have gained content related insights, and have also changed their teaching practice based on the new insights and collectively designed lessons. We have categorized the kinds of learning reported in the studies, while we are aware that the categories can be distinguished, but in teacher learning processes they can often not be separated. We distinguish the following categories of teacher learning: (a) Lesson planning and preparation (including design capacity); (b) Pedagogical Content Knowledge (PCK); (c) Classroom practices; (d) General pedagogy; (e) Social issues and teacher identity in the mathematics classroom (e.g. teaching for equity, identity development). In the following paragraphs we describe each category in more detail.

(a) Lesson planning and preparation (including design capacity)

The focus on lesson planning and preparation appears typical for Lesson study approaches. Participation in Lesson Studies has helped teachers in a Korean and Chinese context to realize the importance of creating detailed lesson plans to accomplish mathematical learning goals. Teachers reported that, in several rounds of lesson study, they learned to anticipate student reasoning and to design tasks that evoked this reasoning (Huang, Gong, & Han, 2016; Pang, 2016). Also in an Australian

study on structured problem-solving primary-school mathematics teachers reported that they had learned to appreciate the value of creating a detailed lesson plan in Japanese lesson study (Groves, Doig, Vale, & Widjaja, 2016). It had made the teachers realize "just how much there is to the teaching and learning when you step back from the actual lesson or class itself" (ibid, pp508). Enacting the lessons and receiving feedback from observers in post-lessons discussions were driving forces for their learning, which had led to changes in their classroom pedagogy. However, the teachers also noted that the Japanese approach was difficult to implement in an Australian classroom due to the different classroom cultures: in the Australian context, small-group rather than whole-class teaching was emphasized.

(b) PCK

Several studies reported PCK-related learning gains, which typically depended on the mathematical topic of the professional development project described in the study. This learning was even relevant for teachers at pre-school level. Thouless and Gifford (2019) studied the learning of UK teachers from 6 schools who participated in a two-year professional development project. The teachers formed a community of practice, in which also researchers were involved. The teachers developed their knowledge of patterns and changed their teaching of this topic by jointly developing pedagogical approaches. Other examples of PCK-related learning gains in teacher collective work at different school levels include: proportional reasoning in the primary school curriculum (Hilton & Hilton, 2019), exploring the functions between two variables by students at middle school level (Wilkie, 2016), implementation of mathematical modeling tasks in middle school (Jung & Brady, 2016), meaningfully integrating the concepts of functions and graphs in combined science / mathematics tasks in upper secondary school (Potari et al., 2016).

Involvement in the design of educational technology can contribute to the development of teachers' technological pedagogical content knowledge. This was demonstrated in a study by Hansen, Mavrikis, and Geraniou (2016) with a group of primary school mathematics specialists in the UK, who co-designed virtual manipulative on fractions and used it in their classrooms. A promising to develop teachers' technological pedagogical content knowledge was described by Misfeldt and Zacho (2016): in their project teachers developed digital learning environments, using GeoGebra and Google sites to create open-ended projects for students, based on the concepts of educational scenarios and games. More research was needed to overcome among others the steep technological learning curve for some participants.

(c) Classroom practices

Some studies on PD projects including teacher collective work aimed at a change of classroom practices, generally with the purpose to move away from mathematics focused on procedures, towards student conceptual understanding and mathematical reasoning. In lesson study projects, classroom practices change as a result of enacting the collectively developed lesson. For example, a Chilean lesson study project focused

on primary teachers developing classroom practices to maintain high cognitive demand (as opposed to procedural or routine efforts) in the implementation of statistic lessons (Estrella, Zakaryan, Olfos, & Espinoza, 2020). In types of PD programs other than lesson studies, teacher collective work was included as an effective way for teachers to learn (e.g. see Wiliam, Lee, Harrison, & Black, 2004). Veldhuis and van den Heuvel-Panhuizen (2020) took this approach when developing PD workshops for primary school teachers in the Netherlands to help them develop classroom assessment techniques, methods that allow the teacher to get a quick overview of students' skills and knowledge of relevant mathematical content, so as to provide meaningful formative feedback. In the workshops, teachers and researchers collaboratively developed classroom assessment techniques, based on mathematical and pedagogical analysis of the mathematical content. Significant increases in student achievement scores on standardized mathematics tests were found. A change of classroom practices was also the purpose of a design research project in New Zealand in which teachers and researchers collaboratively aimed to improve statistics lessons for Pasifika students whose home language was not English (Sharma, 2019).

In Sweden, a large scale professional development program took place in which more than 33,000 mathematics teachers participated (Bergqvist, Liljekvist, van Bommel, & Österholm, 2017). Purpose of the program was, among others, to develop the teaching culture at the schools towards teaching for the development of mathematical competencies in line with a new national curriculum (e.g. problem solving, conceptual understanding, mathematical reasoning, and modelling). The program consisted to a large extent of supervised teacher collaboration and discussions, including the use of web-based support modules. An evaluation study in 35 schools, based on observations and interviews before, during and after the program showed that significant and sustained changes took place in teachers' classroom practices towards the development of mathematical competences. The researchers argue that the program was successful, because "the teachers were given organized possibilities to develop their knowledge and abilities to teach in line with the new curriculum documents" (pp160).

(d) General pedagogy

In several studies, teacher learning was reported that took place in the context of teaching mathematics, but was not typical for mathematics teaching. This learning typically consisted of an increased ability to notice, to reflect on teaching and learning, and to take the perspective of the student. For example, Tan and Lim (2017) studied in Malaysia how the primary teachers' reflections on lessons developed by participating in several rounds of lesson study. They found that with increasing experience in the reflection process, teachers reflected in more detail on student learning, shifted their perspective from the teacher to the students, changed perspective during their reflections and were able to anticipate student responses when refining lesson plans and student tasks. Such a shift took also place in a one year PD program studied by Haßler et al. (2015) with primary school teachers in Zambia, in schools serving

disadvantaged communities. This study aimed to promote interactive forms of subject teaching in conjunction with Open Educational Resources (OER) and technology. An increasing ability to reflect and the development of a reflective language that supports deep discussions about core issues was found in a study on PD project in which Israeli mathematics teachers watched and discussed videotaped lessons of unknown teachers (Karsenty & Arvaci, 2017). Another study on the use of video, a video club for rural mathematics teachers in the USA, reported an increasing ability to notice student thinking, and to use it for instructional decisions (Wallin & Amador, 2019).

Collective work of teachers in PD is not a guarantee for effective teacher learning. Dalby (2021) studied a design research project in which groups of secondary school mathematics teachers in the UK designed lessons to explore the use of iPads for formative assessment. Findings show that teachers made progress towards this aim, both technically and pedagogically. However, comparing two groups, she found that they did not develop as equally effective professional learning communities. Group leadership, how often communication between members took place, and the extent to which group members felt ownership of the aims had an impact on the effectiveness. For individual members, also their prior technical knowledge influenced their learning.

(e) Social issues and teacher identity in the mathematics classroom

Several studies reported teacher learning in terms of in terms of doing justice to students of mathematics and developing their own professional identities as mathematics teachers. In New Zealand, professional development focusing on the collective redesign and enactment of classroom practices in schools serving disadvantaged communities helped mathematics teachers to see more and different mathematical capacities in their students (Hunter et al., 2020). Such developments of supporting student engagement may result in a shift of mathematics teachers' professional identity from knowledge providers to "'facilitators', 'learners' and 'co-creators' of knowledge" (Bobis et al.). Others (e.g. Nicol et al., 2017) used discussion and reflection as to explore possibilities and challenges of teaching mathematics for justice. The process was described as complex, with dialogue, contradictions and discomfort playing a role.

6. Conclusions

In this section we analyze the findings from the previous three sections and link them to our earlier conceptualizations of the theoretical frames (see section 4).

First, we ask 'what means school-based teacher collective work', because we have seen that teacher collective work can happen in person, or at a distance (whether teachers sit at home or in school). Different initiatives for distance collaboration can be activated through MOOCs (e.g., Italy), through platforms (e.g., Denmark, France, Netherlands), or through websites (e.g., Israel). These initiatives ask for a more nuanced description and conceptualization of school-based teacher collaborative work.

Second, this re-conceptualization of teacher learning communities also needs to include the ways these communities are supported: e.g., are teachers given time, as a matter of course, to collaborate at school (or at a distance), or do they have to ask the head teacher to carve out time for such activities? Do teachers have opportunities to meet and discuss their lesson planning? If teachers are expected to participate in such communities (that provide opportunities for discussions with colleague professionals), a culture of collaboration is needed. This is particularly needed, so the literature argues (e.g., Lamb & Visnovska 2012), in rural communities and small schools, where there is a smaller number of mathematics teachers who can support greater collegial collaboration. Leaning on Millet and Bibby's (2004) 'zone of enactment', school-based teacher collective work needs to be supported, for example with low-cost digital resources that allow for video conferencing from different sites. They can also be supported by teachers' working condition: e.g., where the professional learning in school -based communities is counted as a 'normal' daily task of a teacher (as it is in China and Japan). Or they can be supported by individual projects (e.g., as in Europe by EU projects); however, the sustainability of such initiatives is not ensured, and often the learning community 'disconnects' when the project finishes.

Third, we have seen that Lesson Study (albeit in different forms) exists all across the world. However, and according to local or regional or national practices, they are differently 'lived' in different cultures.

Fourth, we have seen that different forms and participation of professional learning communities provide different forms of agency for teachers. It seems that teachers always enact agency, even when they seemingly accept practices from others. These enactments have to be seen in relation to particular social and material conditions in their environment (and often relations of power). More research is needed to understand how teachers enact agency in school, and more particularly in school based collaborative communities.

Fifth, we have seen that teacher learning in collective work takes place in many domains. Teachers may gain competence in preparing their lessons, in mathematical classroom practices, in general pedagogy and in helping their students learn mathematics in more equitable ways. They gain PCK and develop their professional identities as a result of their activities. A common element of many studies is that teachers learn from enacting new practices in the classroom, collectively reflecting on these practices, and developing them further. In many cases knowledgeable others, experts or researchers facilitate the learning processes (and may be the initiators of the professional development programs in which teachers participate). The Lesson Studies in Japan and China are, indeed, part of teachers' regular school practice. However, such initiatives do not exist in all countries and other studies we examined were set up as projects with a limited duration. It can be expected that many mathematics teachers do not take part in these projects, but do participate in collective activity as part of their regular practice. Hence, there is a need to do further research on how these teachers learn and develop professionally.

References

J. M. Amador and I. S. Carter (2018). Audible conversational affordances and constraints of verbalizing professional noticing during prospective teacher lesson study. *Journal of Mathematics Teacher Education*, 21, 5–34. https://doi.org/10.1007/s10857-016-9347-x

T. Baba, A. Ueda, H. Ninomiya, and K. Hino (2018). Mathematics Education Lesson Study in Japan from Historical, Community, Institutional and Development Assistance Perspectives. *Mathematics Lesson Study Around the World*, Springer, 23–45.

T. Bergqvist, Y. Liljekvist, J. van Bommel, and M. Österholm (2017). Evaluation of a large scale professional development program. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 153–160). Singapore: PME.

G. Biesta, M. Priestly, and S. Robinson (2015). The role of beliefs in teacher agency. *Teachers and Teaching: Theory and Practice*, 21 (6), 624–640.

J. Bobis, M. Khosronejad and J. Way, et al. "Sage on the stage" or "meddler in the middle": shifting mathematics teachers' identities to support student engagement. *Journal of Mathematics Teacher Education*, 23, 615–632 (2020). https://doi-org/10.1007/s10857-019-09444-1

H. Borko (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, 33(8), 3–15.

H. Borko, J. Frykhom, M. E. Pittman, E. Eiteljorg, M. Nelson, J. Jacobs ... C. Schneider (2005). Preparing teachers to foster algebraic thinking. *ZDM Mathematics Education*, 37(1), 43–52.

K. Brodie (2021) Teacher agency in professional learning communities, *Professional Development in Education*, 47:4, 560–573, DOI: 10.1080/19415257.2019.1689523

P. Cobb, et al. (2018). *Systems for instructional improvement: creating coherence from the classroom to the district office.* Cambridge: Harvard Education Press.

M. Cochran-Smith and S. Lytle (2001). Beyond certainty: Taking an inquiry stance on practice. In A. Lieberman & L. Miller (Eds.), *Teachers caught in the action: Professional development in practice* (pp. 45–60). New York: Teachers College Press.

D. Dalby (2021). Professional learning through collaborative research in mathematics. *Professional Development in Education*, 47(4), 710–724. doi:10.1080/19415257.2019.1665571

L. Darling-Hammond (1994). Developing professional development schools: Early lessons, challenges, and promise. In L. Darling-Hammond (Ed.), *Professional development schools: Schools for developing a profession* (pp. 1–27). New York: Teachers College Press.

L. Ding and K. Jones (2020). A comparative analysis of different models of mathematics teacher collaboration and learning. In H. Borko & D. Potari (eds.), *ICMI Study 25 Conference Proceedings*: Teachers of mathematics working and learning in collaborative groups. Universidade de Lisboa. pp. 110–117.

L. Ding, K. Jones, and S. A. Sikko (2019). Interconnectedness and difference between action research and a lesson design study in Shanghai, China. *Educational Action Research*, 27(4), 595–612. https://doi.org/10.1080/09650792.2019.1579745

R. DuFour, R. Eaker and R. Dufour (2005). *On Common Ground: The Power of Professional Learning Communities*. Bloomington, IN: Solution Tree.

X. Dumay, T. Boonen, and J.Van Damme (2013). Principal leadership long- term indirect effects on learning growth in mathematics. Elem. *School J.* 114, 225–251. doi: 10.1086/673198

22

S. Estrella, D. Zakaryan, R. Olfos, and G. Espinoza (2020). How teachers learn to maintain the cognitive demand of tasks through Lesson Study. *Journal of Mathematics Teacher Education*, 23, 293–310. doi:10.1007/s10857-018-09423-y

A. Etelapelto, et al. (2013). What is agency? Conceptualising professional agency at work. *Educational Research Review*, 10, 45–65. doi:10.1016/j.edurev.2013.05.001

C. Fernandez and M. Yoshida (2004). *Lesson study: A case of a Japanese approach to improving instruction through school-based teacher development*. Mahwah: Lawrence Erlbaum.

T. Fujii (2016). Designing and adapting tasks in lesson planning: a critical process of Lesson Study. *ZDM Mathematics Education,* 48, 411–423. https://doi.org/10.1007/s11858-016-0770-3

Y. L. Goddard, R. Miller, R. Larsen, G. Goddard, R. Jacob, J. Madsen, et al. (2010). "Connecting principal leadership, teacher collaboration, and student achievement," in Paper presented at the American Educational Research Association Annual Meeting (Denver, CO).

P. Graham (2007). Improving teacher effectiveness through structured collaboration: a case study of a professional learning community. Res. *Middle Level Educ*. 31, 1–17. doi: 10.1080/19404476.2007.1 1462044

S. Groves, B. Doig, C. Vale, and W. Widjaja (2016). Critical factors in the adaptation and implementation of Japanese Lesson Study in the Australian context. *ZDM Mathematics Education*, 48, 501–512. doi:10.1007/s11858-016-0786-8

F. Gu and L. Gu (2016). Characterizing mathematics teaching research specialists' mentoring in the context of Chinese lesson study. *ZDM Mathematics Education*, 48, 441–454.

A. Hansen, M. Mavrikis, and E. Geraniou (2016). Supporting teachers' technological pedagogical content knowledge of fractions through co-designing a virtual manipulative. *Journal of Mathematics Teacher Education*, 19, 205–226. doi:10.1007/s10857-016-9344-0

B. Haßler, S. Hennessy, A. Cross, E. Chileshe, and B. Machiko (2015). School-based professional development in a developing context: lessons learnt from a case study in Zambia. *Professional Development in Education*, 41(5), 806–825.

J. Hattie (2003). "Teachers make a difference: what is the research evidence?" In Paper presented at the Australian Council for Educational Research Annual Conference on Building Teacher Quality (Melbourne, VIC). Retrieved from: http://research.acer.edu.au/research_conference_2003/4/

J. Hattie (2015). *What Works Best in Education: The Politics of Collaborative Expertise*. London: Pearson.

L. R. Herrenkohl, K. Kawasaki, and L. S. Dewater (2010). Inside and outside: Teacher–researcher collaboration. *The New Educator*, 6(1), 74–92.

A. Hilton and G. Hilton (2019). Primary school teachers implementing structured mathematics interventions to promote their mathematics knowledge for teaching proportional reasoning. *Journal of Mathematics Teacher Education*, 22, 545–574. doi:10.1007/s10857-018-9405-7

J. Hochweber, B. Steinert, and E. Klieme (2012). *Lehrerkooperation, Unterrichtsqualität und Lernergebnisse im Fach Englisch*. [The impact of teacher cooperation and instructional quality on learning in English as a foreign language]. Unterrichtswissenschaft 40, 351–370.

I. S. Horn, B. D. Kane, and B. Garner (2018). Teacher collaborative time: helping teachers make sense of ambitious teaching in the context of their schools (pp. 93–112). In: P. Cobb, et al. (Eds). *Systems for instructional improvement: creating coherence from the classroom to the district office*. Cambridge: Harvard Education Press.

R. Huang, Z. Gong, and X. Han (2016). Implementing mathematics teaching that promotes students' understanding through theory-driven lesson study. *ZDM Mathematics Education*, 48, 425-439. doi:10.1007/s11858-015-0743-y

I. Isoda (2020). Producing Theories for Mathematics Education Through Collaboration: A Historical Development of Japanese Lesson Study (pp. 15–22). In H. Borko & D. Potari (Eds.) *Conference Proceedings of ICMI Study 25*. Lisbon, Portugal.

C. K. Jackson and E. Bruegmann (2009). Teaching students and teaching each other: the importance of peer learning for teachers. *Am. Econ. J.* 1, 85–108. doi: 10.3386/w15202

R. Johnson, S. Severance, and W. R. Penuel, et al. (2016). Teachers, tasks, and tensions: lessons from a research–practice partnership. *Journal of Mathematics Teacher Education*, 19, 169–185. https://doi.org/10.1007/s10857-015-9338-3

H. Jung and C. Brady (2016). Roles of a teacher and researcher during in situ professional development around the implementation of mathematical modeling tasks. *Journal of Mathematics Teacher Education*, 19, 277–295. doi:10.1007/s10857-015-9335-6

R. Karsenty and A. Arvaci (2017). Mathematics, lenses and videotapes: a framework and a language for developing reflective practices of teaching. *Journal of Mathematics Teacher Education*, 20, 433–455. doi:10.1007/s10857-017-9379-x

R. Karsenty, Y. Peretz, and E. Heyd-Metzuyanim (2019). From judgmental evaluations to productive conversations: Mathematics teachers' shifts in communication within a video club (pp. 3400–3408). *Eleventh Congress of the European Society for Research in Mathematics Education*, Utrecht University, Feb 2019, Utrecht, Netherlands. hal-0242256.

K. Krainer and C. Spreitzer (2020). Collaborative Groups in Mathematics Teacher Education: Grasping the Diversity of Roles, Identities and Interactions (pp.23–26). In H. Borko & D. Potari (Eds.) *Conference Proceedings of ICMI Study 25*. Lisbon, Portugal.

J. Lamb and J. Visnovska (2012). Developing a culture of collaboration. (pp. 1–8). In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), Mathematics education: Expanding horizons. *Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia)*. Singapore: MERGA. © Mathematics Education Research Group of Australasia Inc.

J. Larsen and P. Liljedahl (2020). Public musings about mathematics teaching: social responsibility and ideational alignment. In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.), *Interim Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education* (pp. 313–322). PME.

S. M. C. Lau and S. Stille (2014). Participatory research with teachers: toward a pragmatic and dynamic view of equity and parity in research relationships. *European Journal of Teacher Education*, 37(2), 156–170.

C. Lewis (2016). How does lesson study improve mathematics instruction? *ZDM Mathematics Education*, 48, 571–580. https://doi.org/10.1007/s11858-016-0792-x

M. L. Lo and F. Marton (2012). Towards a science of the art of teaching: Using variation theory as a guiding principle of pedagogical design. *International Journal of Lesson and Learning Studies*, 1, 7–22.

C. Manolino (2020). The semiosphere lens to look at lesson study practices in their cultural context: A case study. In M. Inprasitha, N. Changsri, & N. Boonsena (Eds.), *Interim Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education* (pp. 359–367). PME.

S. Mercer (2011). Understanding learner agency as a complex dynamic system. *System*, 39, 427–436. doi:10.1016/j. system.2011.08.001

M. Misfeldt and L. Zacho (2016). Supporting primary-level mathematics teachers' collaboration in designing and using technology-based scenarios. *Journal of Mathematics Teacher Education*, 19, 227–241. doi:10.1007/s10857-015-9336-5

A. Millett and T. Bibby (2004). The context for change. In A. Millett, M. Brown & M. Askew (Eds.), *Primary mathematics and the developing professional* (pp. 1–17). Netherlands: Kluwer Academic Publishers.

A. Murata (2011). Introduction: Conceptual Overview of Lesson Study. In L. C. Hart et al. (Eds.). *Lesson Study Research and Practice in Mathematics Education*, 1 DOI 10.1007/978-90-481-9941-9_1, © Springer Science+Business Media B.V.

C. Nicol, L. A. Bragg, V. Radzimki, T. He, and K. Yaro (2017). Teaching mathematics for justice: pedagogies of discomfort, contradictions and dialogue. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 297–304). PME.

J. Pang (2016). Improving mathematics instruction and supporting teacher learning in Korea through lesson study using five practices. *ZDM Mathematics Education*, 48, 471–483. doi: 10.1007/s11858-016-0768-x

B. Pepin, G. Gueudet, and L. Trouche (2017). Refining teacher design capacity: Mathematics teachers' interactions with digital curriculum resources. *ZDM Mathematics Education*, 49(5), 799–812. DOI 10.1007/s11858-017-0870-8

B. Pepin, B. Xu, L. Trouche, and C. Wang (2017). Developing a deeper understanding of mathematics teaching expertise: an examination of three Chinese mathematics teachers' resource systems as windows into their work and expertise. *Educational Studies in Mathematics*, 94 (3), 257–274. Doi:10.1007/s10649-016-9727-2

D. Potari, G. Psycharis, V. Spiliotopoulou, C. Triantafillou, T. Zachariades, T, and A. Zoupa, (2016). Mathematics and science teachers' collaboration: searching for common grounds. In C. Csíkos, A. Rausch, & J. Szitányi (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 91–98). Szeged, Hungary: PME.

L. D. Puchner and A. R. Taylor (2006). Lesson study, collaboration and teacher efficacy: stories from two school-based math lesson study groups. *Teaching and Teacher Education*, 22, 922–934. doi: 10.1016/j.tate.2006. 04.011

S. Sharma (2019). Addressing language barriers in multilingual statistics classrooms: A collaborative study. In G. Hine, S. Blackley, & A. Cooke (Eds.), *Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia* (pp. 668–675). Perth: MERGA.

M. G. Sherin, K. A. Linsenmeier, and E. A. van Es (2009). Issues in the design of video clubs: Selecting video clips for teacher learning. *Journal of Teacher Education*, 60(3), 213–230.

C. K. Skott and H. Moeller (2020). Adaptation of Lesson Study in a Danish context: Displacements of teachers' work and power relations. *Teaching and Teacher Education*, 87. https://doi.org/10.1016/j.tate.2019.102945

J. W. Stigler and J. Hiebert (2016). Lesson study, improvement, and the importing of cultural routines. *ZDM Mathematics Education*, 48, 581–587. https://doi.org/10.1007/s11858-016-0787-7

A. L. Tamborg (2021). Improving mathematics teaching via digital platforms? Implementation processes seen through the lens of instrumental genesis. *ZDM Mathematics Education*. https://doi.org/10.1007/s11858-021-01282-x

S. F. Tan and C. S. Lim (2017). Enchancing teachers' reflection through lesson study: is it feasible? In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the*

41st Conference of the International Group for the Psychology of Mathematics Education (Vol. 4, pp. 257–264). Singapore: PME

E. Taranto, O. Robutti, and F. Arzarello (2020). Learning within MOOCs for mathematics teacher education. *ZDM Mathematics Education*, 52, 1439–1453. https://doi.org/10.1007/s11858-020-01178-2

H. Thouless and S. Gifford (2019). Paper plate patterns: Pre-school teachers working as a community of practice. In M. Graven, H. Venkat, A. Essien, & P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 375–381). Pretoria, South Africa: PME.

M. Veldhuis and M. van den Heuvel-Panhuizen (2020). Supporting primary school teachers' classroom assessment in mathematics education: effects on student achievement. *Mathematics Education Research Journal*, 32, 449–471. doi:10.1007/s13394-019-00270-5

N. C. Verhoef, et al. (2015). Professional development through Lesson Study: teaching the derivative using GeoGebra. *Professional Development in Education*, 41 (1), 109–126.10.1080/19415257.2014.886285

N. C. Verhoef, et al. (2013). The complexities of Lesson Study in a European situation. *International Journal of Science and Mathematics Education*, 12 (4), 1–23.

P. Wang-Iverson and M. Yoshida (2005). Building our understanding of lesson study. Philadelphia: Research for Better Schools.

A. J. Wallin and J. M. Amador (2019). Supporting secondary rural teachers' development of noticing and pedagogical design capacity through video clubs. *Journal of Mathematics Teacher Education*, 22, 515–540. doi:10.1007/s10857-018-9397-3

D. Wiliam, C. Lee, C. Harrison, and P. J. Black (2004). Teachers developing assessment for learning: impact on student achievement. *Assessment in Education: Principles, Policy and Practice*, 11, 49–65.

K. J. Wilkie (2016). Learning to teach upper primary school algebra: changes to teachers' mathematical knowledge for teaching functional thinking. *Mathematics Education Research Journal*, 28, 245–275. doi:10.1007/s13394-015-0151-1

Y. D. Yang (2009). "How a Chinese Teacher Improved Classroom teaching in Teaching Research Group: A case study on Pythagoras Theorem teaching in Shanghai", *ZDM (The International Journal on Mathematics Education)*, Vol. 41. No.3. pp. 279–296.

Y. D. Yang and T. E. Ricks (2013). Chinese lesson study: Developing classroom instruction through collaborations in school-based teaching research group activities. In Y. P. Li, & R. J. Huang (Eds.), *How Chinese teach mathematics and improve teaching* (pp. 51–65). New York, NY: Routledge.